Bayesian Approach for CKM Fits

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Inferential Method

Bayesian CKM Fits in a nutshell :

Given N parameters x_i (A, B_K, f_{B_d},....) and M constraints c_j (Δm_d, ϵ_K,...) (whose actual value depends on x_i and on CKM parameters $(\bar{\rho}, \bar{\eta})$) what is the best determination of $(\bar{\rho}, \bar{\eta})$? Bayes Theorem tell us that:

$$f(\bar{\rho}, \bar{\eta}, x_1, x_2, ..., x_N | \hat{c}_1, \hat{c}_2, ..., \hat{c}_M) \propto \prod_{j=1,M} f_j(\hat{c}_j | \bar{\rho}, \bar{\eta}, x_1, x_2, ..., x_N) \prod_{j=1,M} f_i(x_i) \times f_o(\bar{\rho}, \bar{\eta})$$

where f is the p.d.f. for the constraints or parameters and f_o is the a-priori probability for $(\bar{\rho}, \bar{\eta})$

The output p.d.f. for $(\bar{\rho}, \bar{\eta})$ is obtained by integrating over the parameters space:

$$f(\bar{\rho}, \bar{\eta}) \propto \int \prod_{j=1,M} f_j(\hat{c}_j | \bar{\rho}, \bar{\eta}, x_1, x_2, ..., x_N) \prod_{j=1,M} f_i(x_i) \times f_o(\bar{\rho}, \bar{\eta}) dx_i$$

Inferential Method (cont'd.)

Several remarks are in order:

- The present knowledge on each quantity is expressed with a p.d.f. or, if you prefer, with a Likelihood function (in the bayesian approach is always possible to define a p.d.f. using $f(x) \propto \mathcal{L}(x) f_o(x)$)
- The method does not make any distinction (formally) between theoretical and experimental parameters or between gaussian or non-gaussian likelihood.
  The method can easily digest any kind of p.d.f./Likelihood (at variance with other methods).
- The bayes concept of "updating" the knowledge is naturally applied. You start with no information on $(\bar{\rho}, \bar{\eta})$ (constant a-priori) and you update it (improve it) using experimental and theoretical information.

Inputs

How do we treat the inputs ?

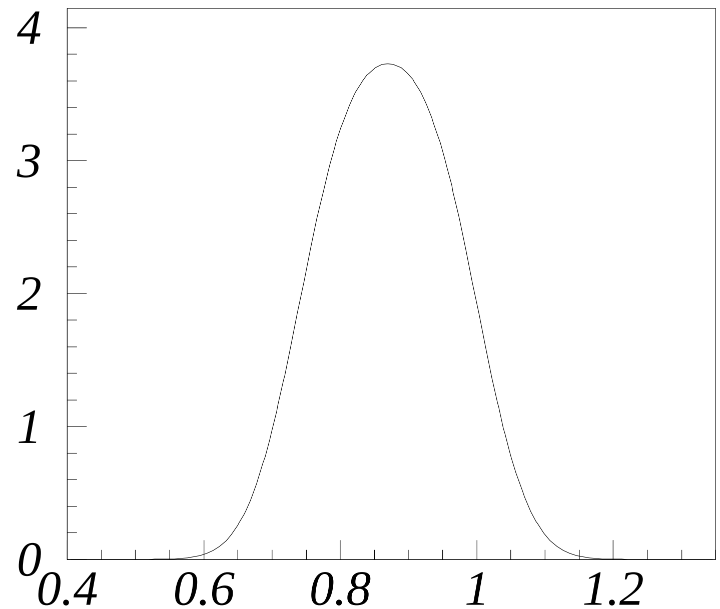
- Where available we take the experimental likelihoods: gaussian measurements, Δm_s.
- If a parameter has an uncertainty given as "allowed range" we assume this parameter to be uniformly distributed in the range. This could apply in principle both to a theoretical estimate or to an experimental systematics.

The final p.d.f. of the parameter is computed as a convolution between all the uncertainties.

Inputs (cont'd.)

p.d.f. for a theoretical parameter:

$$B_K = (0.87 \pm 0.06_{stat} \pm 0.13_{flat})$$



- The most relevant (but unavoidable) assumption is on the size of the range (flat part) of the theoretical uncertainty, not in the shape of the p.d.f.
- The central value is preferred (reasonable).

Inputs (cont'd.)

- The main criticism to this approach is

"You cannot attribute any statistical meaning to the theoretical uncertainty"

This statement is not based on first principles. We can reverse it:

"You should attribute statistical meaning to the uncertainties of any parameters"

This obviously has a certain degree of arbitrariness but obliges to express quantitavely your knowledge. Moreover if we take seriously the first statement we cannot almost talk about errors with statistical meaning !
(several experimental systematics have the same problem)

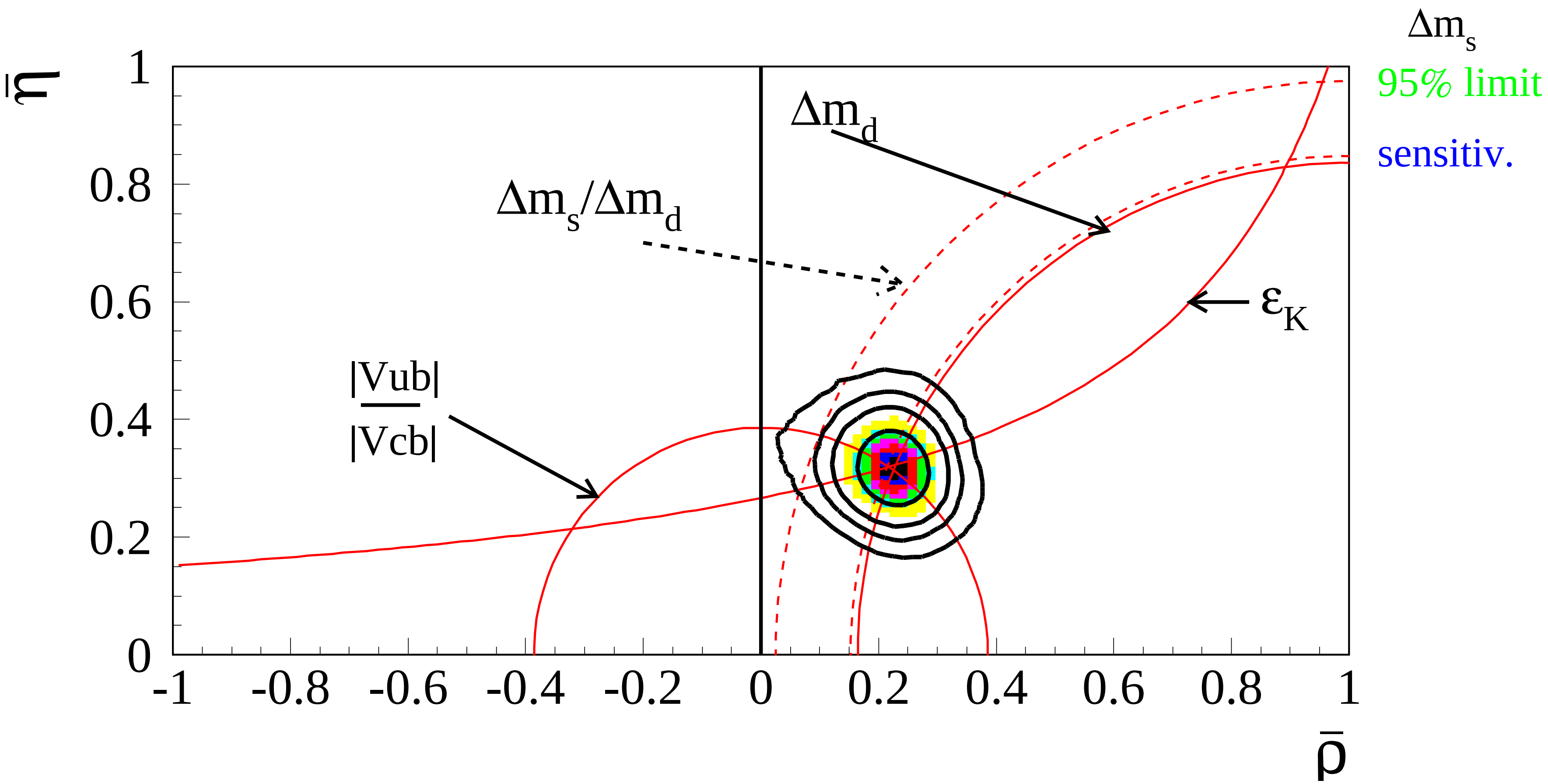
Outputs

The bayesian approach gives outputs with a well defined probabilistic interpretation.

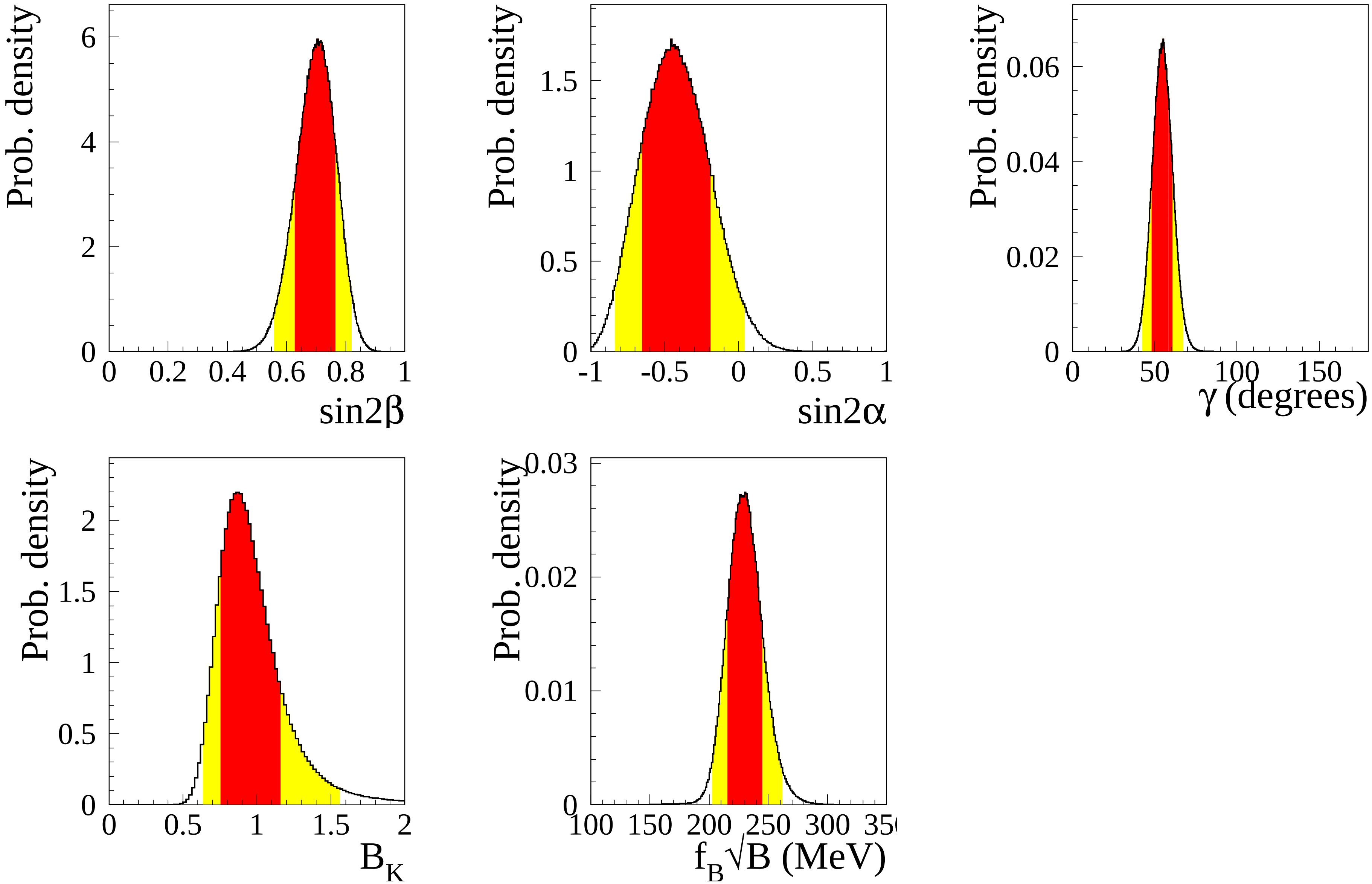
The outputs of a bayesian fit are p.d.f.'s.

- **The allowed regions are well defined in term of probability**. Allowed regions at 95% mean that you expect the "true" value in this range with 95% probability (and not "at least 95%" as, by definition, in the frequentist approaches).
- Any p.d.f. can be extract by changing the integration variables $\hookrightarrow$ **indirect determination of any interesting quantity** (theoretical parameter, unmeasured quantities)

Typical Output



Typical Output (cont'd.)



Outputs (cont'd.)

Compatibility among individual constraints

In the standard χ^2 minimisation the compatibility is evaluated from the value of the χ^2 at its minimum.

Crude arguments based only on the minimum of the χ^2 and the number of degrees of freedom lead to arbitrary conclusions (which is the probability value to claim a disagreement ?).

The alternative procedure, in the bayesian approach, is to remove, in turn, each constraint and compute its expected distrution.
The compatibility of each constraint is then obtained by comparing its measured value and this indirect determination.

Bayesian Summary

Closest to the physicist's way of reasoning.
Raise your hand if you have never summed in quadrature a systematical error !

p.d.f. for each value in input

▼

quantitavely well defined output (p.d.f's !)

And, finally, it gives statements that you can falsify. Just because it gives well defined statements..