

THE UNITARITY TRIANGLE ANALYSIS AND ITS THEORETICAL INPUT PARAMETERS



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OUTLINE

1. Flavour physics and its motivations
2. Lattice QCD and the UTA input parameters
3. UTA: results and perspectives

Orsay,
April 14-16 2004



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FLAVOUR PHYSICS AND ITS MOTIVATIONS

STANDARD MODEL AND FLAVOUR PHYSICS

Elementary Particles

Quarks	u	c	t	Force Carriers	γ
	d	s	b		g
Leptons	ν_e	ν_μ	ν_τ		Z
	e	μ	τ		W

Three Generations of Matter

Electromagnetic

Strong

Weak

$$SU(2)_L \times U(1)_Y \times SU(3)$$

Symmetry breaking:
Higgs (?)

FLAVOUR PHYSICS

- Why 3 families?
- Why the hierarchy of masses?
- Is the CKM mechanism and its explanation of CP violation correct?

Flavour

Only weak interactions can change flavour [CKM]

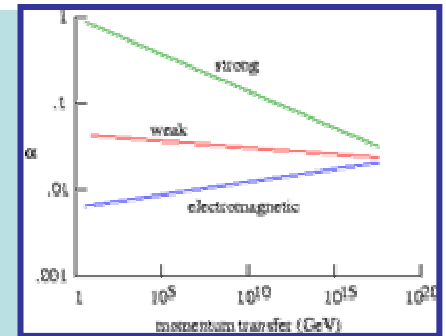
THE STANDARD MODEL: A LOW ENERGY EFFECTIVE THEORY

CONCEPTUAL PROBLEMS The most obvious:

o Gravity: $M_{\text{Planck}} = (\hbar c/G_N)^{1/2} \approx 10^{19} \text{ GeV}$

PHENOMENOLOGICAL INDICATIONS:

- o Unification of couplings ($M_{\text{GUT}} \approx 10^{15}-10^{16} \text{ GeV}$)
- o Dark matter ($\Omega_M \approx 0.35$)
- o Neutrino masses
- o Matter/Anti-matter asymmetry (not enough \cancel{CP} in the SM)

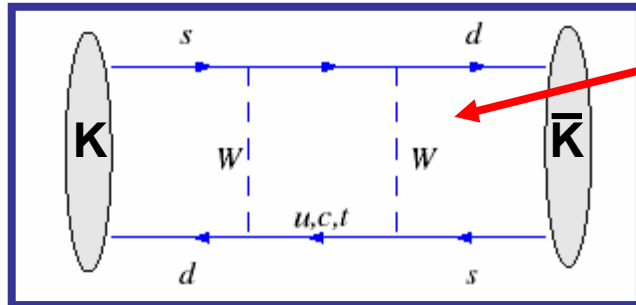


THE "NATURAL" CUT-OFF: $\delta m_H^2 = \frac{3G_F}{\sqrt{2}\pi} m_t^2 \Lambda^2 \approx (0.3 \Lambda)^2$

→ $\Lambda = O(1 \text{ TeV})$

NEW PHYSICS MUST BE
VERY "SPECIAL"

NEW PHYSICS EFFECTS IN FLAVOUR PHYSICS



QUANTUM EFFECTS

THE FLAVOUR PROBLEM:

$$\Lambda_{K^0-\bar{K}^0} \approx O(100 \text{ TeV})$$

PRECISION ERA OF FLAVOUR PHYSICS

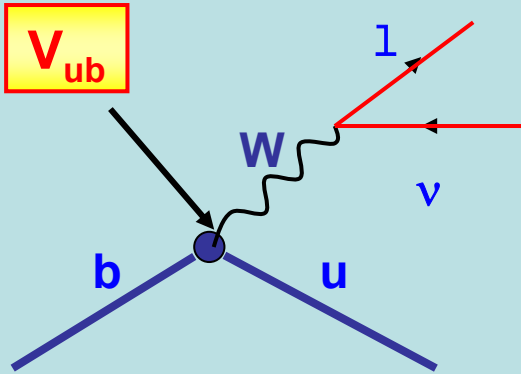
$$\epsilon_K = (2.271 \pm 0.017) \times 10^{-3} \quad 0.7\%$$

$$\Delta m_d = (0.503 \pm 0.006) \text{ ps}^{-1} \quad 1\%$$

$$\sin(2\beta) = 0.734 \pm 0.054 \quad 7\%$$

We need to control the theoretical input parameters at a comparable level of accuracy !!

THE CKM MATRIX



$$L_W = -\frac{g}{2\sqrt{2}} V_{ij} \bar{u}_i \gamma^\mu W_\mu^+ (1 - \gamma^5) d_j + \text{h.c.}$$

3 FAMILIES:

3 angles and **1** phase

Only 1 parameter for
CP VIOLATION

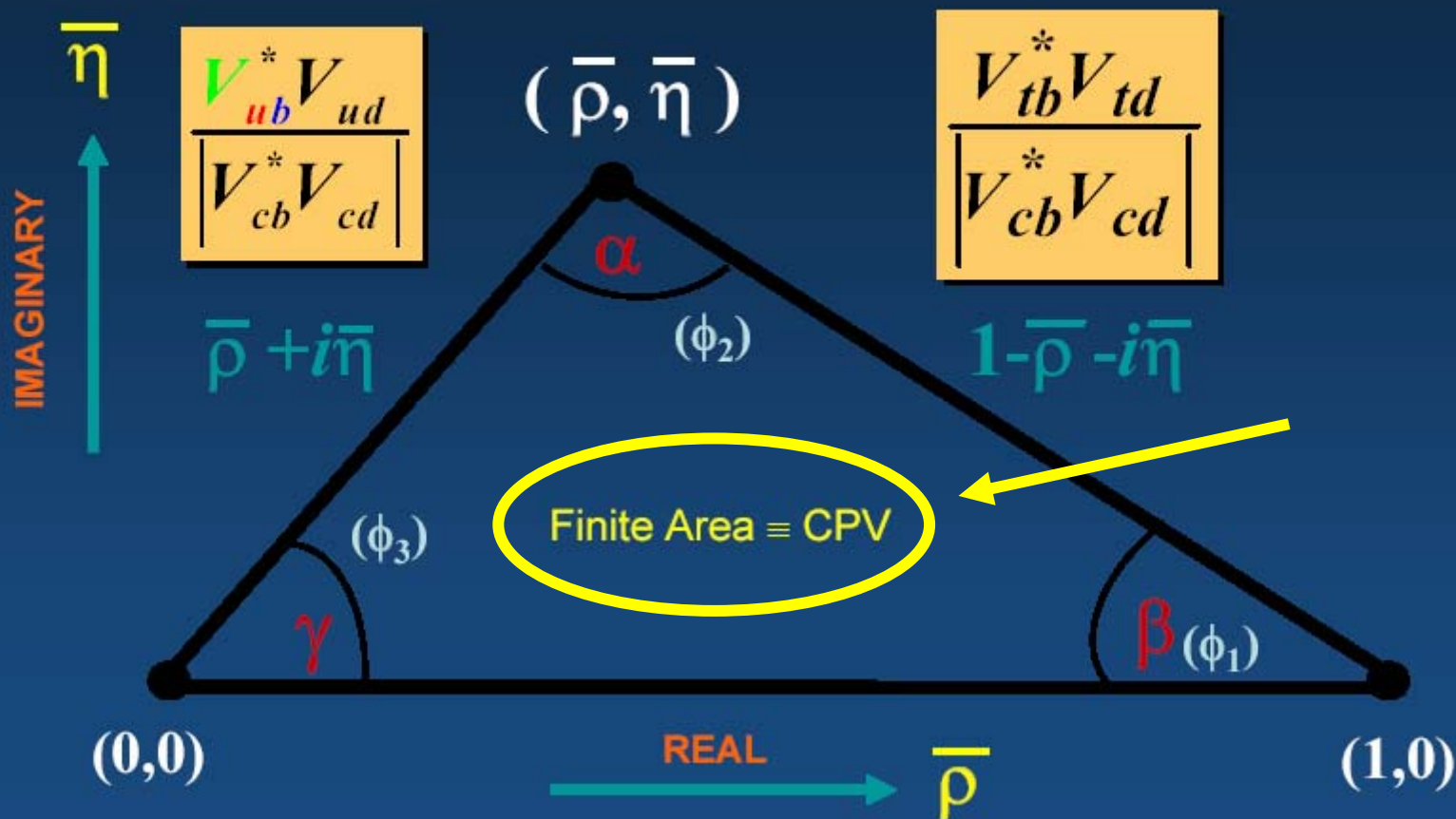
$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \approx \begin{bmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4)$$

(Lincoln Wolfenstein)

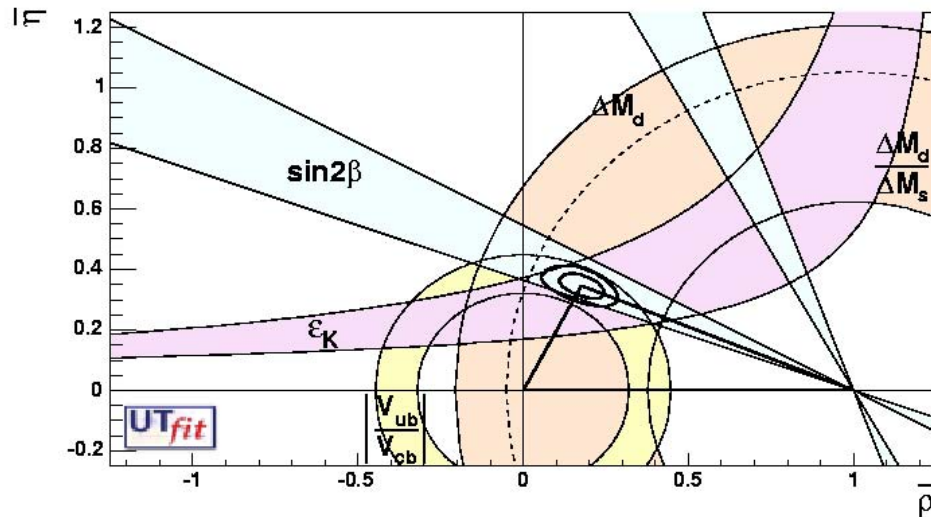
THE UNITARITY TRIANGLE

Unitarity:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



THE UNITARITY TRIANGLE ANALYSIS



5 CONSTRAINTS
2 PARAMETERS

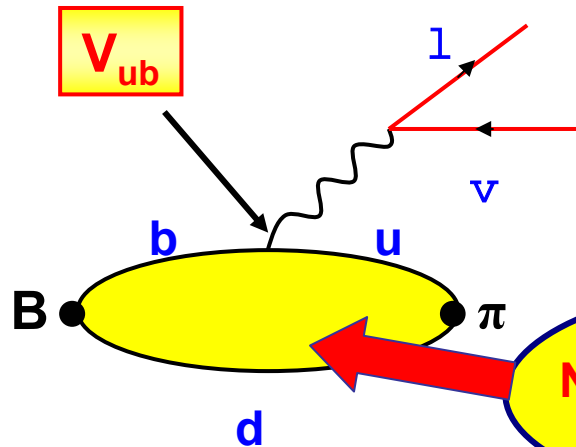
Hadronic Matrix
Elements from
LATTICE QCD

$(b \rightarrow u)/(b \rightarrow c)$	$\bar{\rho}^2 + \bar{\eta}^2$	$\bar{\Lambda}, \lambda_1, f_+, \dots$
ϵ_K	$\bar{\eta} [(1 - \bar{\rho}) + P]$	B_K
Δm_d	$(1 - \bar{\rho})^2 + \bar{\eta}^2$	$f_{B_d}^2 B_{B_d}$
$\Delta m_d / \Delta m_s$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$	ξ
$A(J/\psi K_S)$	$\sin 2\beta(\bar{\rho}, \bar{\eta})$	—

2

LATTICE QCD AND THE UTA INPUT PARAMETERS

V_{ub} FROM B-MESON SEMILEPTONIC DECAYS

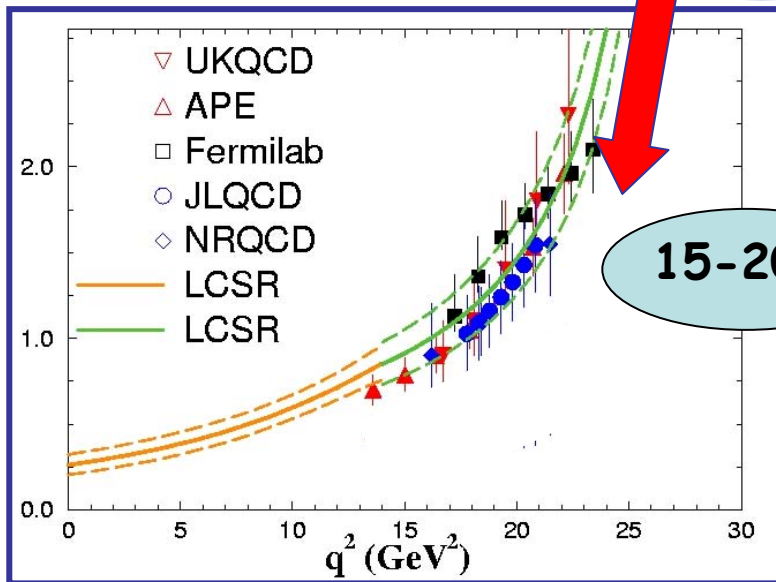


$$\Gamma(B \rightarrow \pi l \bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192 \pi^3} \int dq^2 \lambda(q^2)^{3/2} |f_+(q^2)|^2$$

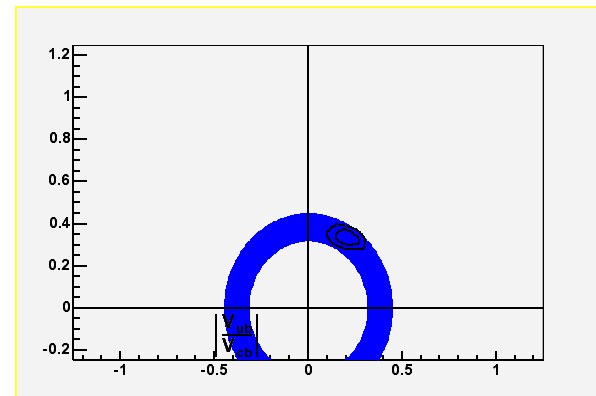
NON-PERTURBATIVE PHYSICS

EXPERIMENTS:

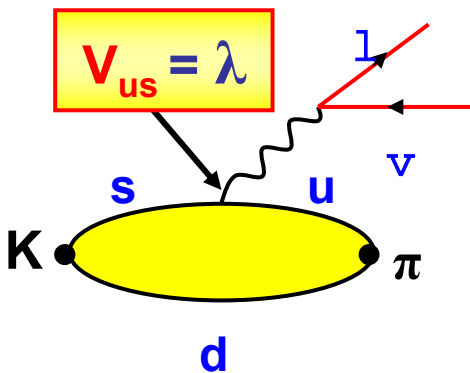
CLEO, BaBar, Belle, ...



15-20%

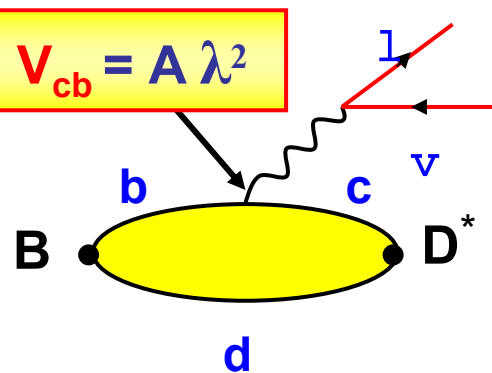


PRECISION FLAVOUR PHYSICS ON THE LATTICE



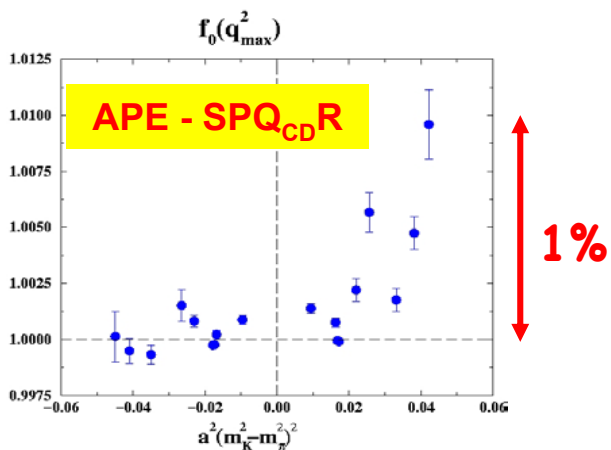
$$f_+(0) = 1 - O(m_s - m_u)^2$$

Ademollo-Gatto theorem

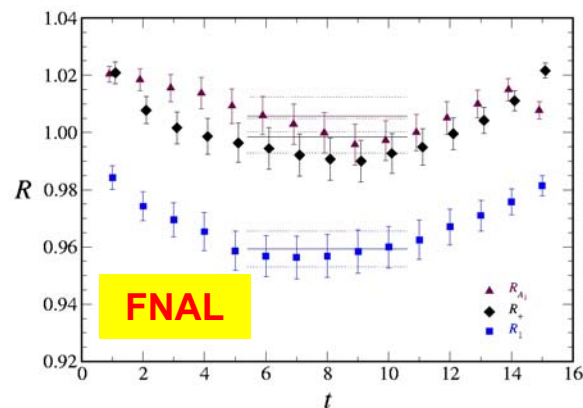


$$F_{B \rightarrow D^*}(1) = \eta_A [1 - O(1/m_b, 1/m_c)^2]$$

Luke theorem



$$f_+(0) = 0.960 \pm 0.005 \pm 0.007$$

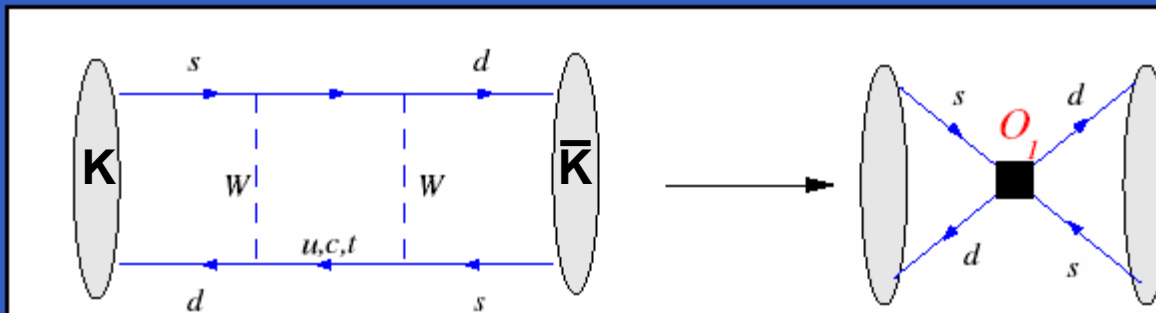
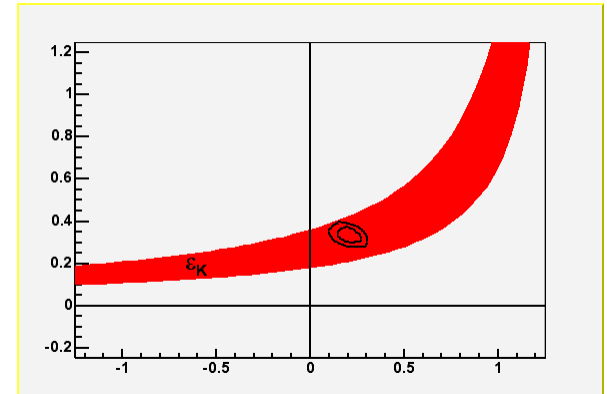


$$F_{B \rightarrow D^*}(1) = 0.913^{+0.024}_{-0.017} \pm 0.017 \pm 0.030$$

K – \bar{K} Mixing: ϵ_K and B_K

$$K_L \sim \overset{\text{CP}=-1}{(K^0 - \bar{K}^0)} + \epsilon_K \overset{\text{CP}=+1}{(K^0 + \bar{K}^0)}$$

CP Violation



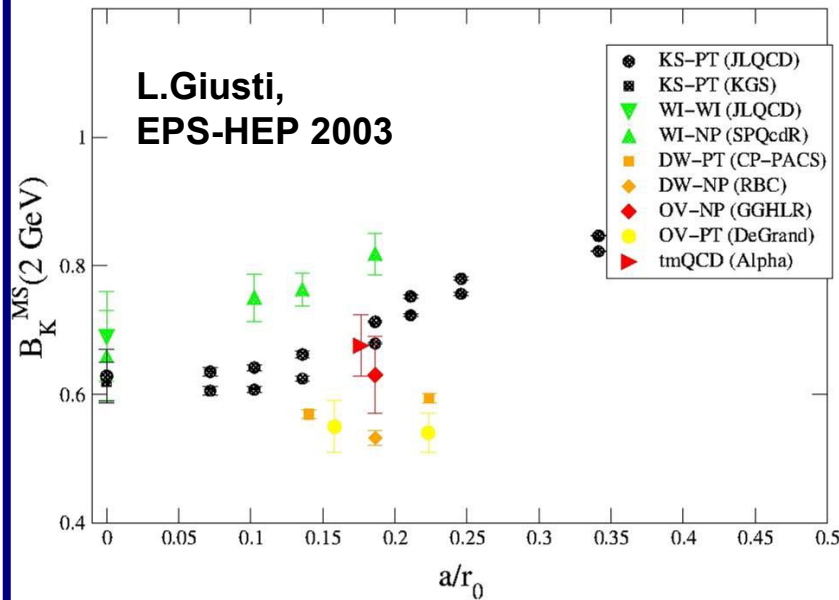
NON-PERTURBATIVE PHYSICS

$$\epsilon_K \sim \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle$$

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

Lattice Results for B_K

B_K from Quenched QCD

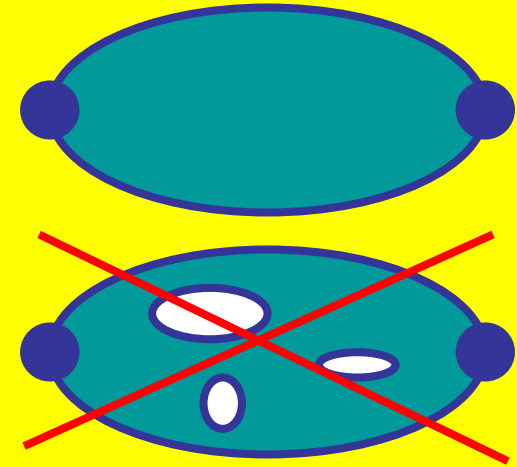


- ✓ High level of accuracy
- ✓ Discretization effects not negligible
- ✓ Estimate of quenching error from ChPT $\leq 15\%$ (Sharpe)

Quench. Appr.

$$\hat{B}_K = 0.87 \pm 0.06 \pm 0.13$$

[D. Becirevic, Plenary talk @ LATT03]



LATTICE PREDICTION (!)

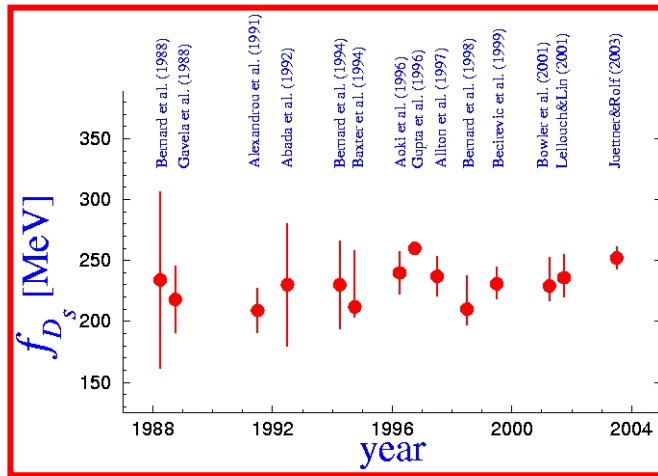
$$\hat{B}_K = 0.90 \pm 0.20$$

[Gavela et al., 1987]

$B_{B_d/s} - \bar{B}_{B_d/s}$ Mixing: $f_{B_d/s}$ and $B_{B_d/s}$

THE D_s -MESON DECAY CONSTANT A long history of lattice calculations...

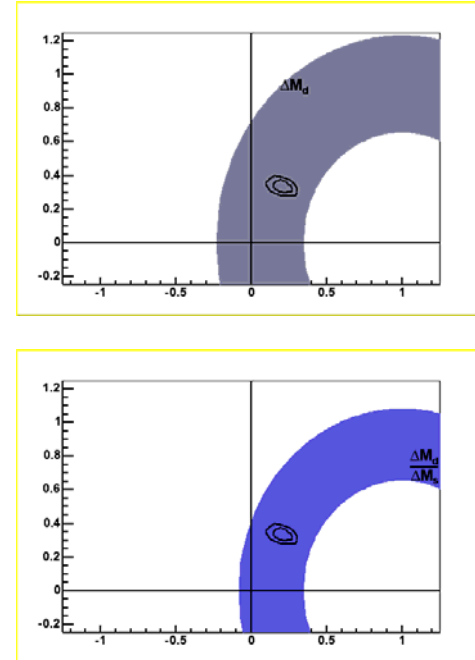
QUENCHED



$$f_{D_s}^{N_f=2} / f_{D_s}^{N_f=0} = 1.08 \pm 0.05 \quad (\text{CP-PACS, MILC})$$

$$f_{D_s} = 265 \pm 14 \pm 13 \text{ MeV} \quad \text{LQCD Average [D. Becirevic]}$$

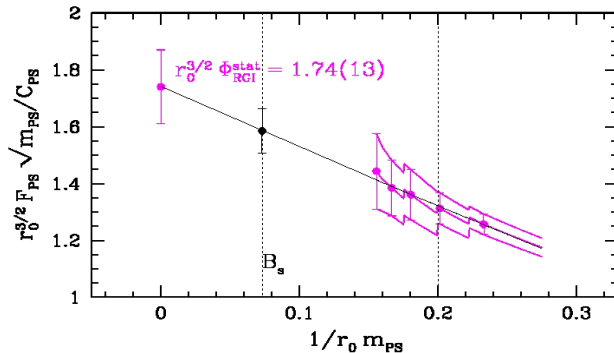
$$f_{D_s} = 285 \pm 19 \pm 40 \text{ MeV} \quad \text{EXP. PDG 2002}$$



New results expected from CLEO-c

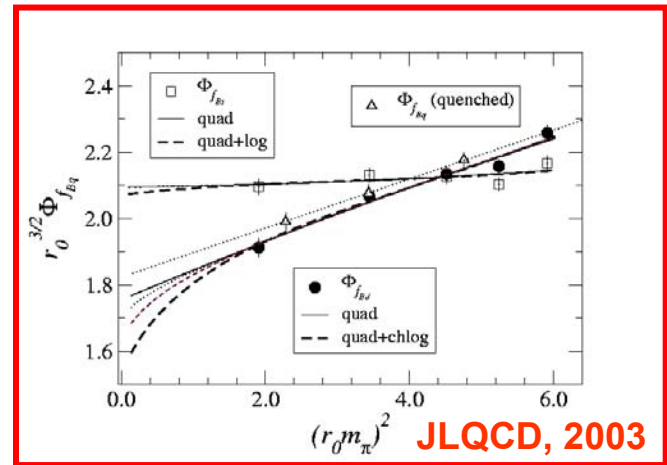
From f_{D_s} to f_{B_d}

J.Heitger, EPS-HEP 2003



$$m_Q \ll 1/a$$

- Extrapolations from $m_Q \sim m_c$ to m_b
- Effective theories: HQET, NRQCD, "FNAL", ...
- Combine the two approaches
- Finite size approach, APE-Tov



$$m_\pi \gg 1/L$$

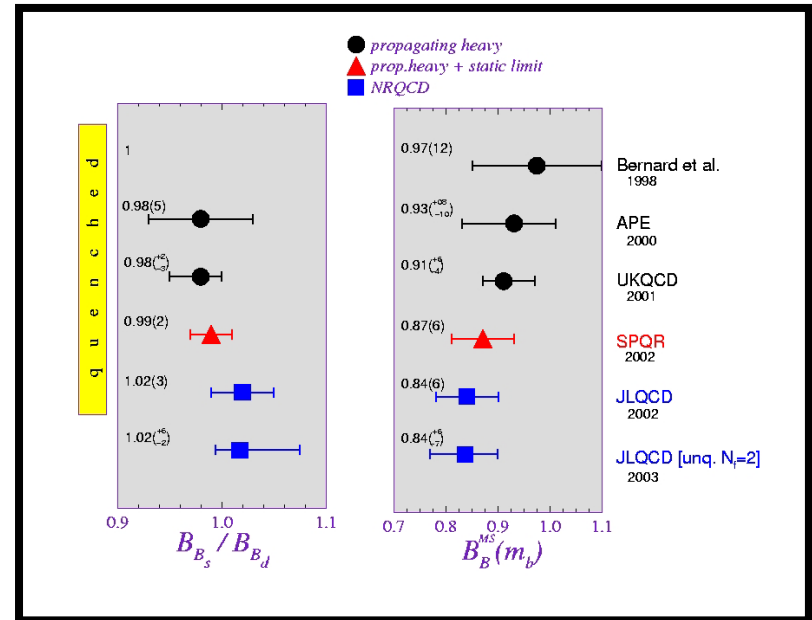
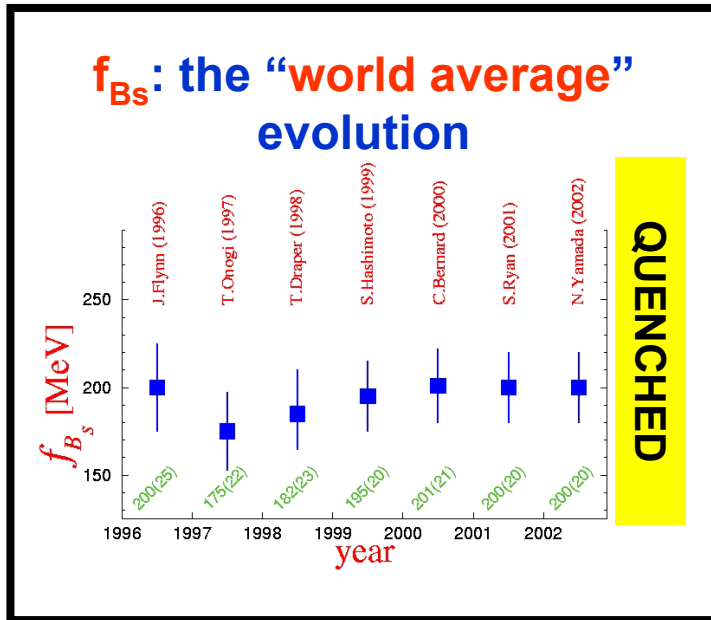
- Extrapolations from m_q to $m_{u,d}$ using ChPT as a guidance:

$$\frac{f_{B_s} \sqrt{m_{B_s}}}{f_{B_d} \sqrt{m_{B_d}}} = 1 + \frac{1 + 3\hat{g}^2}{4(4\pi f)^2} \chi \log s + C$$

Orsay

Use $\frac{f_{B_s}/f_{B_d}}{f_K/f_\pi}$, Becirevic et al.

$B_{B_{d/s}} - \bar{B}_{B_{d/s}}$ Mixing: $f_{B_{d/s}}$ and $B_{B_{d/s}}$



$$f_{B_s}^{N_f=2} / f_{B_s}^{N_f=0} = 1.12 \pm 0.05$$

(CP-PACS, MILC)

In other quantities (f_{B_s} / f_{B_d} , B_{B_d} , B_{B_s} / B_{B_d}) quenching effects are smaller

**LATTICE
AVERAGES**

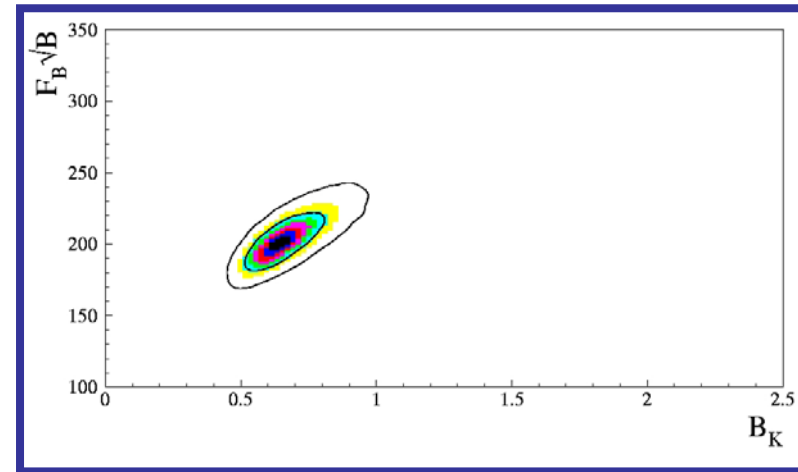
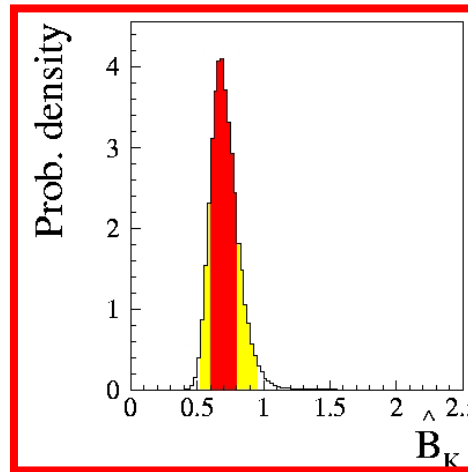
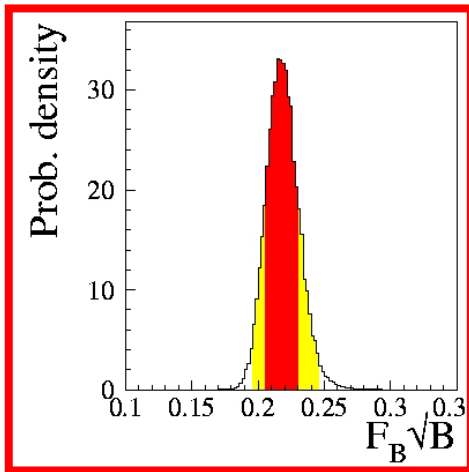
$$f_{B_s} \sqrt{B_{B_s}} = 276 \pm 38 \text{ MeV}, \quad \xi = 1.24 \pm 0.04 \pm 0.06$$

[1st Workshop on the CKM Unitarity Triangle, 2002]

$$f_{B_s} \sqrt{B_{B_s}} = 254 \pm 24 \text{ MeV}, \quad \xi = 1.21 \pm 0.05 \pm 0.01$$

[D. Becirevic, 2nd Workshop on the CKM Unitarity Triangle, 2003]

Lattice QCD vs UT FITS



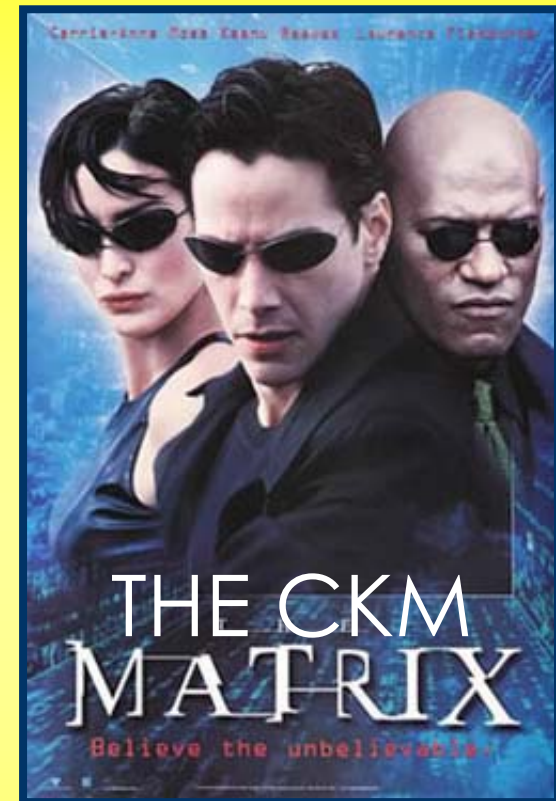
	LATTICE QCD	UT FIT
$f_B \sqrt{B_B}$	$223 \pm 33 \pm 12 \text{ MeV}$	$217 \pm 12 \text{ MeV}$
B_K	$0.86 \pm 0.06 \pm 0.14$	0.71 ± 0.11

3

**THE UNITARITY
TRIANGLE ANALYSIS:
RESULTS AND
PERSPECTIVES**

The Collaboration

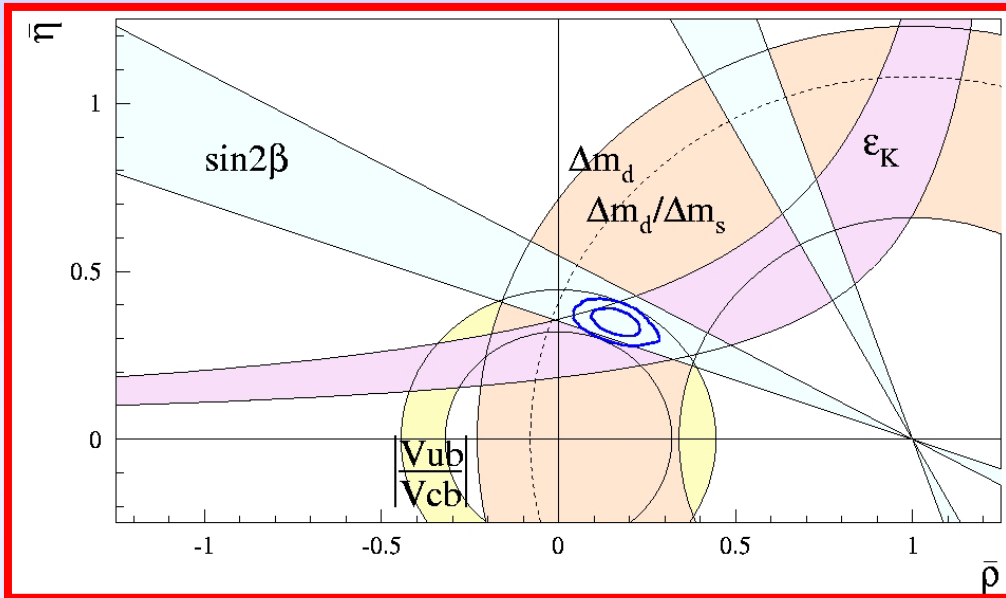
M.Bona, M.Ciuchini, E.Franco,
V.L., G.Martinelli, F.Parodi,
M.Pierini, P.Roudeau, C.Schiavi,
L.Silvestrini, A.Stocchi
Roma, Genova, Torino, Orsay



The WEB site

www.utfit.org

FIT RESULTS



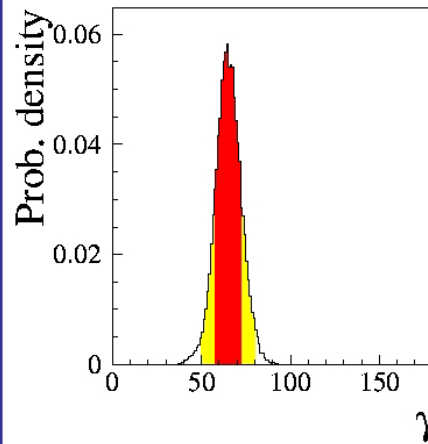
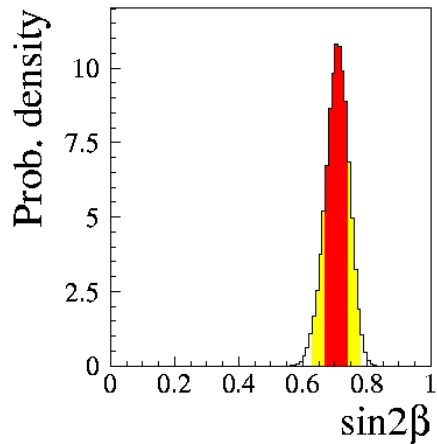
$$\bar{\rho} = 0.178 \pm 0.046$$

$$\bar{\eta} = 0.341 \pm 0.028$$

$$\sin 2\alpha = -0.19 \pm 0.25$$

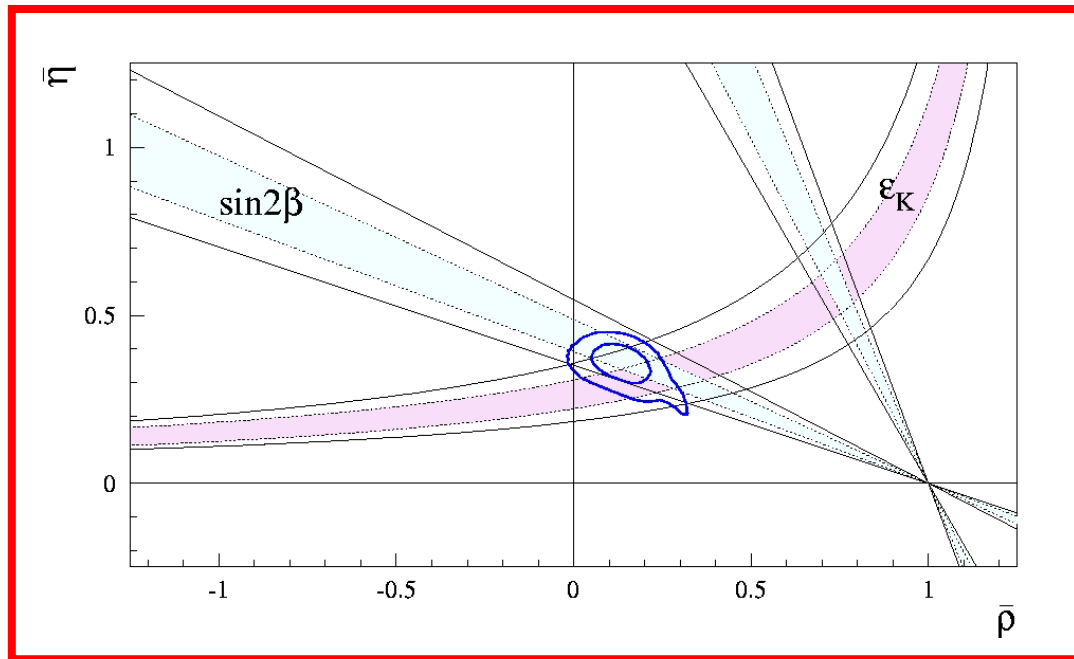
$$\sin 2\beta = 0.710 \pm 0.037$$

$$\gamma = (61.5 \pm 7.0)^\circ$$



INDIRECT EVIDENCE OF CP VIOLATION

3 FAMILIES \rightarrow - Only 1 phase - Angles from Sides



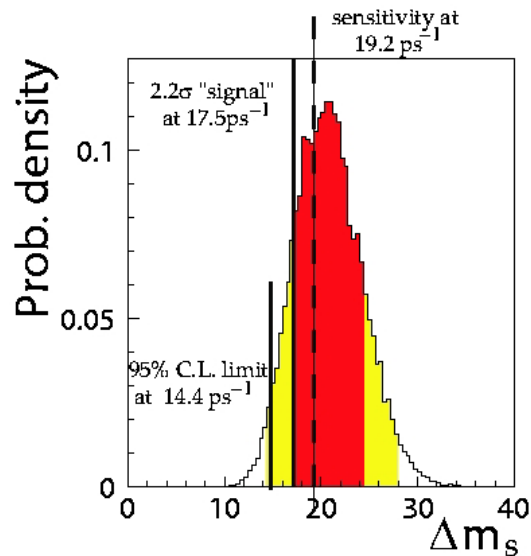
$$\text{Sin}2\beta_{\text{UTA}} = 0.685 \pm 0.052$$

$$\text{Sin}2\beta_{J/\psi \text{ K}s} = 0.734 \pm 0.054$$

Prediction (Ciuchini et al., 2000): $\text{Sin}2\beta_{\text{UTA}} = 0.698 \pm 0.066$

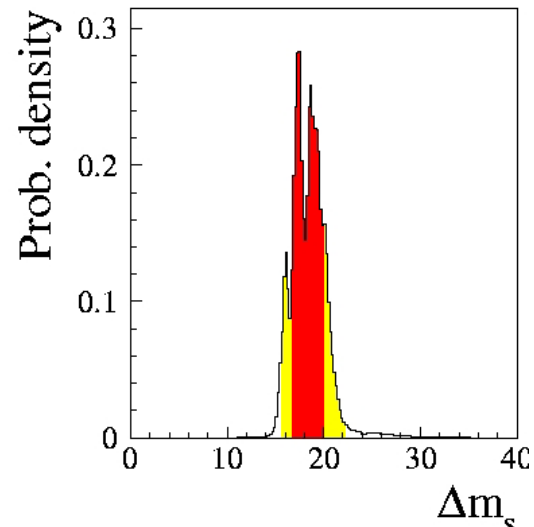
Prediction for Δm_s

Δm_s NOT USED



$$\Delta m_s = (20.6 \pm 3.5) \text{ ps}^{-1}$$

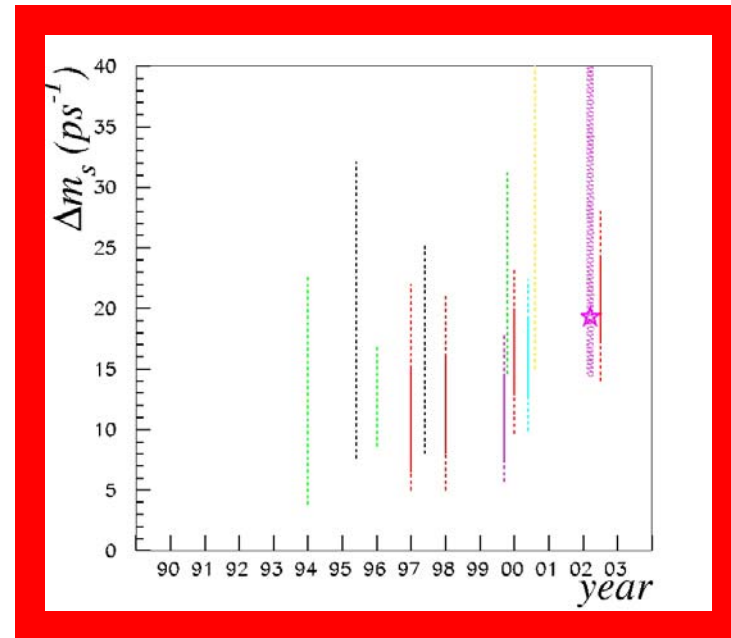
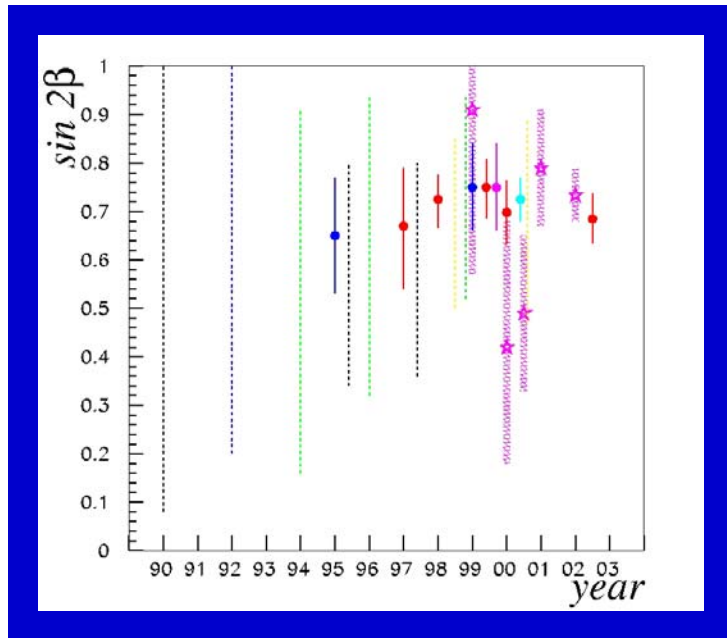
WITH ALL CONSTRAINTS



$$\Delta m_s = (18.3 \pm 1.7) \text{ ps}^{-1}$$

A measurement is expected from Tevatron !

Sin2 β AND Δm_s : HISTORY OF PREDICTIONS

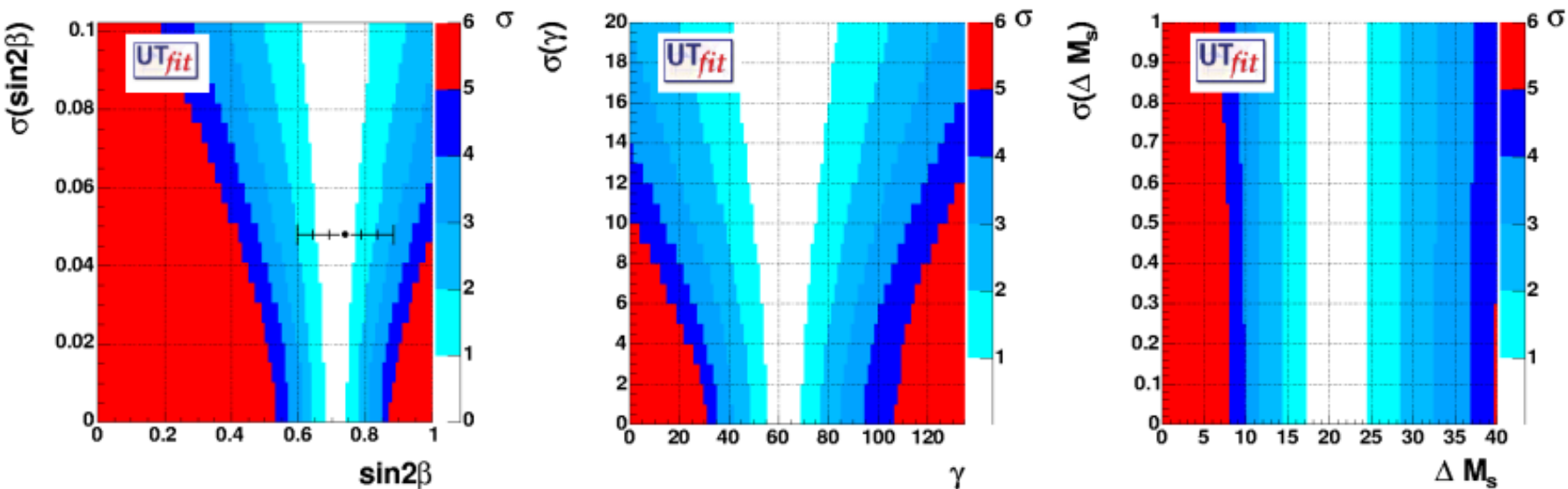


Experiments

Other colors: Theory

THE “COMPATIBILITY” PLOTS

“To which extent improved determinations of the experimental constraints will be able to detect **New Physics?**”



Compatibility between **direct** and **indirect determinations** as a function of the measured value and its experimental uncertainty

IMPACT OF IMPROVED DETERMINATIONS

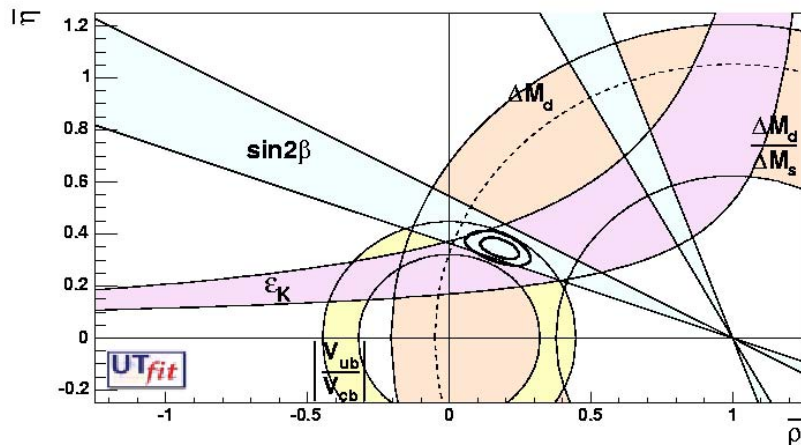
$$B_K = 0.86 \pm 0.06 \pm \cancel{0.14}$$

$$\xi = 1.24 \pm 0.04 \pm \cancel{0.06}$$

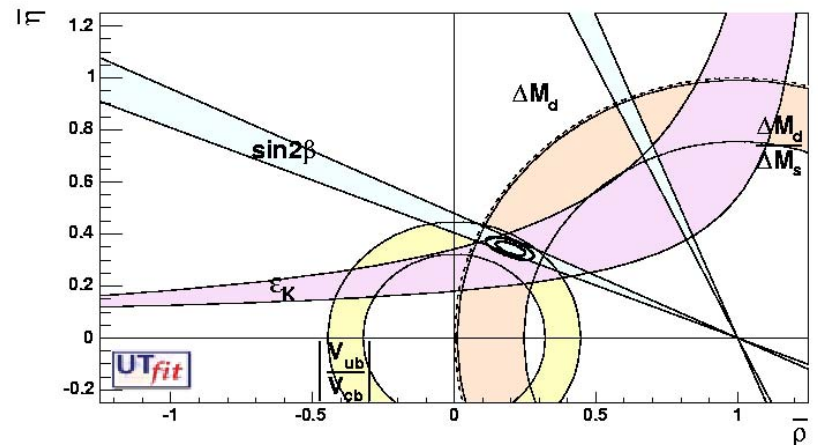
$$f_{B_s} \sqrt{B_{B_s}} = 276 \pm \textcircled{38}^{14} \text{ MeV}$$

$$\sin 2\beta = 0.734 \pm 0.0\textcircled{54}^{21}$$

TODAY

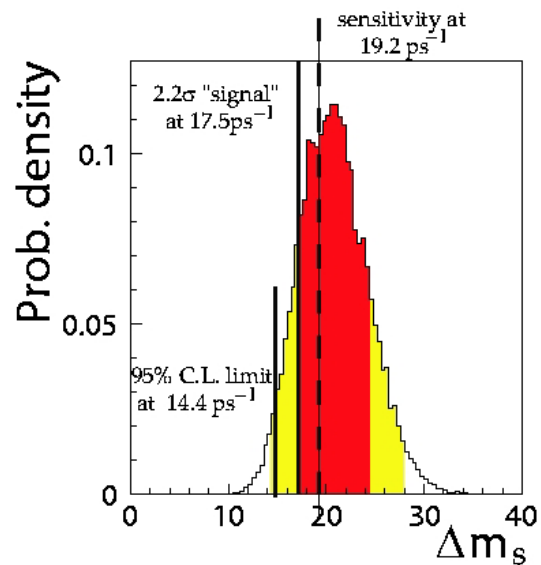


NEXT YEARS

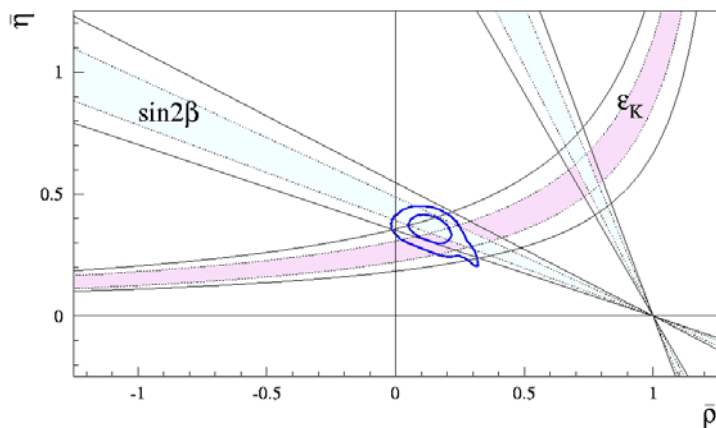


$$\Delta\rho = 26\% \rightarrow 15\% \quad \Delta\eta = 7.1\% \rightarrow 4.6\%$$

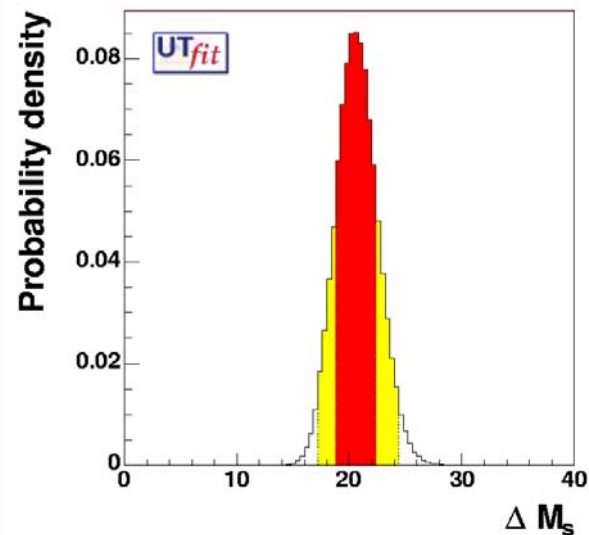
TODAY



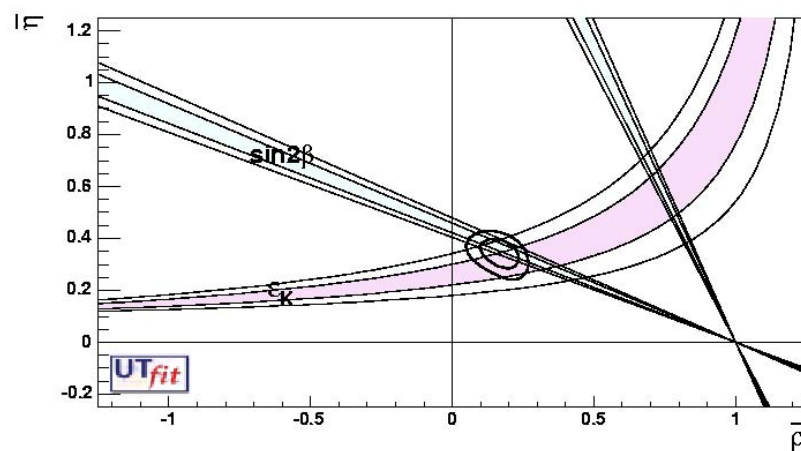
$$\Delta m_s = (20.6 \pm 3.5) \text{ ps}^{-1}$$



NEXT YEARS



$$\Delta m_s = (20.6 \pm 1.8) \text{ ps}^{-1}$$



SEARCH FOR NEW PHYSICS

“Given the present theoretical and experimental constraints, to which extent the UTA can still be affected by **New Physics** contributions?”

An interesting case:

New Physics in $B_d-\bar{B}_d$ mixing

The New Physics mixing amplitudes can be parameterized in a simple general form:

$$M_d = C_d e^{2i\varphi_d} (M_d)^{\text{SM}}$$



$$\Delta m_d = C_d (\Delta m_d)^{\text{SM}}$$
$$A(J/\psi K_S) \sim \sin 2(\beta + \varphi_d)$$

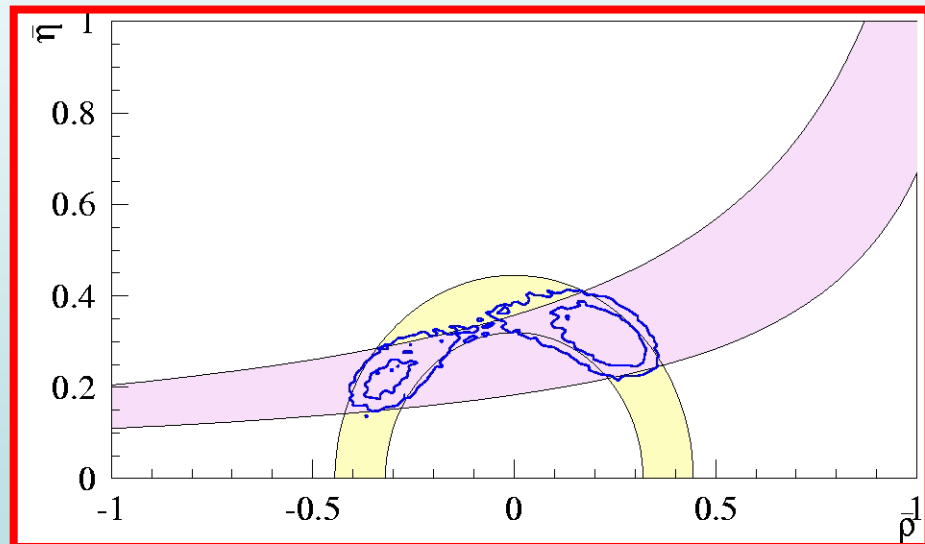
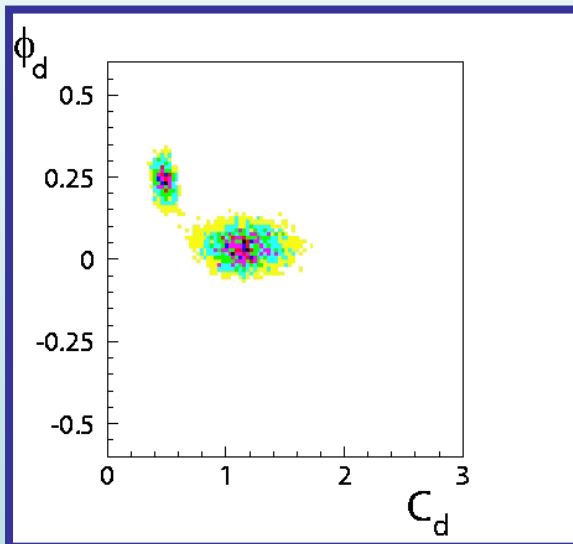
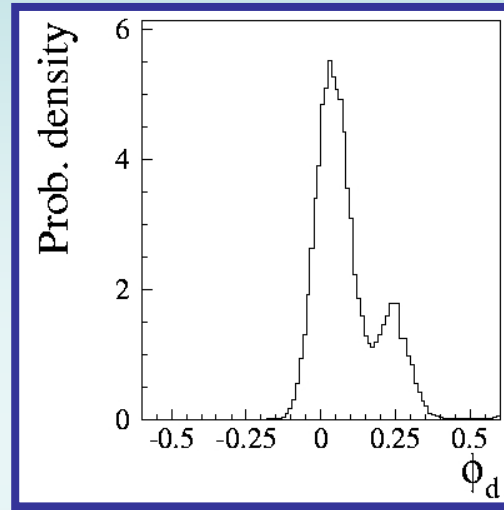
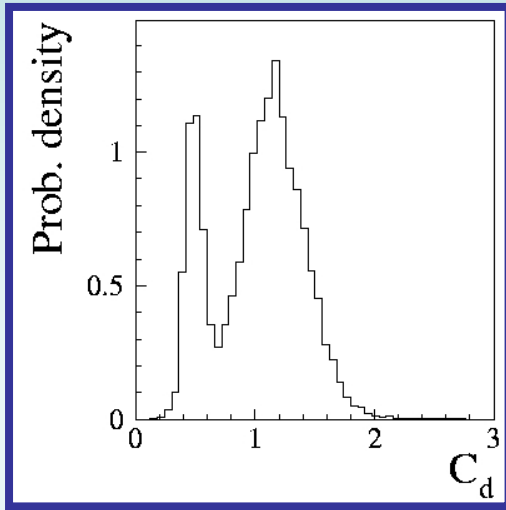
TWO SOLUTIONS:

**Standard Model
solution:**

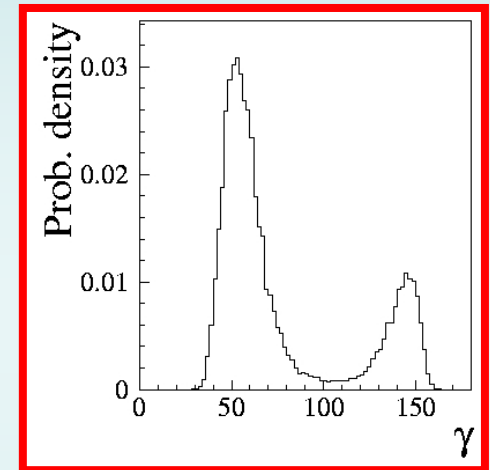
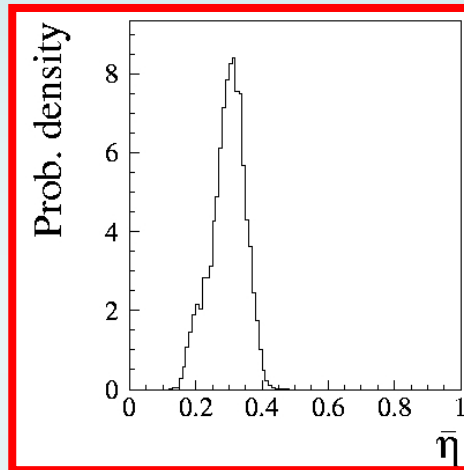
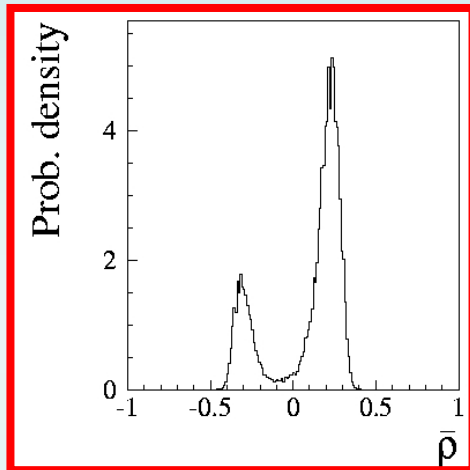
$$C_d = 1 \quad \varphi_d = 0$$

φ_d can be only determined up
to a trivial twofold ambiguity:

$$\beta + \varphi_d \rightarrow \pi - \beta - \varphi_d$$



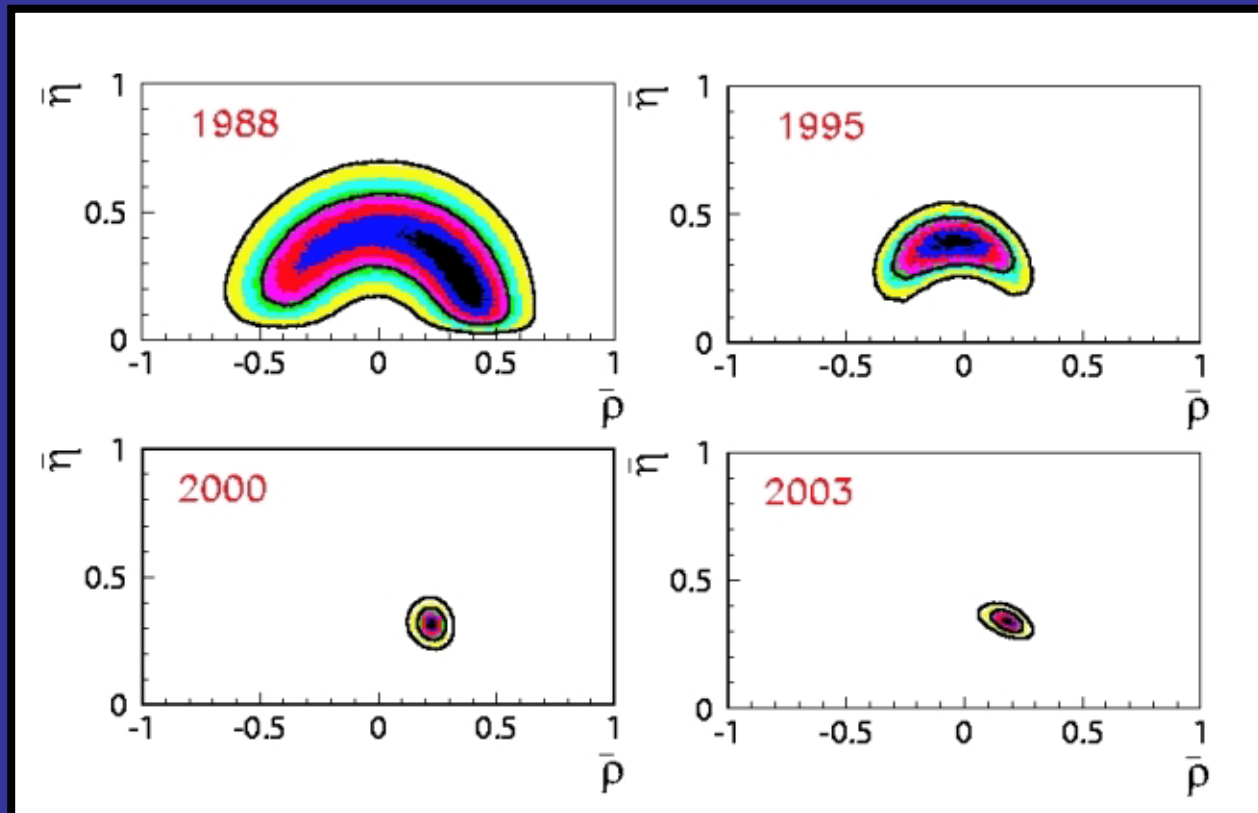
HOW CAN WE DISCRIMINATE BETWEEN THE TWO SOLUTIONS?



~~$\Delta m_s, \eta$~~ [$K_L \rightarrow \pi \nu \bar{\nu}$], γ [$B \rightarrow DK$], $|V_{td}|$ [$B \rightarrow \rho \gamma$], ...

Coming back to the Standard Model:

15 YEARS OF ($\bar{\rho}$ - $\bar{\eta}$) DETERMINATIONS (The “commercial” plot)



CONCLUSIONS

- **LATTICE QCD CALCULATIONS** HAD A CRUCIAL IMPACT ON TESTING AND CONSTRAINING THE FLAVOUR SECTOR OF THE STANDARD MODEL
- IN THE **PRECISION ERA OF FLAVOUR PHYSICS**, LATTICE SYSTEMATIC UNCERTAINTIES MUST (AND CAN) BE FURTHER REDUCED
- IMPORTANT, BUT **MORE DIFFICULT PROBLEMS** (NON LEPTONIC DECAYS, RARE DECAYS, ...) ARE ALSO BEING ADDRESSED

**FOR ALL THAT, WE NEED
T-FLOPS COMPUTERS!!**